

# A formal characterization of relevant information in multi-agent systems

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## Abstract

*This paper studies cooperation in multi-agent systems. In particular, we focus on the notion of relevance underlying cooperation. Given an agent who has some information need, we characterize pieces of information that are relevant to it. This characterization is done in a multi-modal logic. Finally, we give a formal definition for cooperation.*

**Keywords:** cooperation, information relevance, multi-agent systems, information need

## 1 Introduction

This work deals with multi-agent systems in which agents are grouped in order to achieve a global goal none of them would be able to achieve alone. Such a multi-agent system can be a group of robots whose aim is to localize a fire and extinguish it, or a system of systems for search and rescue operation.

In such systems, agents have to cooperatively act so that the achievement of their individual goals ensures the achievement of the global goal. In particular, agents have to cooperatively communicate, which means first that exchanged information is easily understandable by the agent that receives it, i.e. it is expressed in a language it understands and its interpretation does not require too long time nor effort. But more, this implies that *the exchanged information is the very one useful for the receiver to fulfil its current individual goal*. More precisely, the achievement of the agent's individual goal generates some information needs i.e., requires the acquisition of some information we call *relevant information*. For instance, a robot may decide to go to the north direction of the zone (individual goal), and for doing so, it needs to know if the way is free or not (information need). Any true information about the presence of obstacle on the way will be relevant to the robot.

More generally, relevance is a crucial concept in any process of information demanding. For instance, as mentioned by [1], in intelligence gathering for crisis management, intelligence analysts face large amounts of information, provided by several sources of different types and need tools for getting only relevant and reliable information.

Providing a definition of relevant information is thus needed.

Existing definitions of relevance can be found in different domains. According to Borlund [2], definitions of relevance can be separated into two different groups : system-oriented relevance and agent-oriented relevance.

*System-oriented* approaches analyze relevance in terms of topicality, aboutness, matching degrees between a piece of information and a request, or in terms of dependence. Most of the literature belonging to

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this approach can be found in: Information Retrieval where, given a request, the Information Retrieval system finds in its collection of documents the ones relevant for the request [3, 4]; Artificial Intelligence where several notions of relevance have been introduced in order to speed up the inference in knowledge bases [5, 6]; Relevant Logics where alternatives to material implication have been defined so that antecedent and consequent are relevantly related [7].

On the other hand, *agent-oriented* approaches try to define a relation between some agent and a piece of information. Thus, relevance is analyzed in terms of agent's utility or informativeness for the agent. In those cases, relevant pieces of information are defined according the information need of the agent. The literature on this approach is quite informal but is of great interest. In Information Retrieval, Borlund [2] and Mizzaro [8] give a classification of different agent-oriented relevance depending the considered user level. They also point out the main concepts on which relevance is based on such as: the information need of the agent, her background knowledge, the context she is in, etc. In Linguistic, Grice [9] expounds his cooperation principle along with the corresponding maxims. One of the maxim is the relevance maxim and stipulates that one should be relevant in order to be cooperative. Many studies have followed Grice's [10, 11]. In particular, Sperber and Wilson reduce all the Grice's maxims to one and define a cognitive psychological theory, the Relevance Theory, based on the following informal definition: *An input (a sight, a sound, an utterance, a memory) is relevant to an individual when it connects with background information he has available to yield conclusions that matters to him.* Finally, in Philosophy, Floridi [12] has developed a subjectivist interpretation of epistemic relevance. In Floridi's theory, *the degree of relevance of a piece of information I towards an agent A is defined as a function of the accuracy of I understood by A as an answer to a query Q, given the probability that Q might be asked by A.*

In this paper, our aim is to contribute to the study of relevance by giving a formal definition of agent-oriented relevance and studying its properties. Based on this definition, we will then formally define what is cooperation in information exchanging.

This paper is organized as follows. Section 2 presents the multi-modal logic framework we base our model on. Section 3 deals with relevance defined according to an agent's information need. In section 4, we define a hierarchy that characterizes the most relevant pieces of information. In section 5, we propose a characterization for cooperation between agents which exchange information. Finally, section 6 concludes this paper.

## 2 Formal framework

The formalism we use here to model agents and their mental attitudes is a propositional multi-modal logic. The mental attitudes we are interested in are belief and intention. This framework is very close to what has been suggested in [13].

The alphabet of our language is based on non logical symbols : a set  $\mathcal{A}$  of agents, for every agent  $a$  of  $\mathcal{A}$ , we define two modalities  $B_a$  and  $I_a$ . We define also the set of logical symbols : a set  $\mathcal{V}$  of variables symbols,  $\neg$ ,  $\vee$ , ( and ), the constants  $\top$  and  $\perp$ .

**Definition 1** *The formulae of our language are defined recursively as follows:*

- if  $p$  belongs to  $\mathcal{V}$  then  $p$  is a formula of our language.  $\perp$  and  $\top$  are formulae of our language.
- if  $a$  is an agent of  $\mathcal{A}$  and  $\varphi$  a formula of our language then  $B_a\varphi$  and  $I_a\varphi$  are formulae of our language.  $B_a\varphi$  is read "agent  $a$  believes that  $\varphi$  is true".  $I_a\varphi$  is read "agent  $a$  intends  $\varphi$  to be true".
- if  $\varphi_1$  and  $\varphi_2$  are formulae of our language, so are  $\neg\varphi_1$  and  $\varphi_1 \vee \varphi_2$ .

If  $\varphi_1$  and  $\varphi_2$  are formulae of our language and  $a$  some agent of  $\mathcal{A}$ , we also define the following abbreviations:  $\varphi_1 \wedge \varphi_2 \equiv \neg(\neg\varphi_1 \vee \neg\varphi_2)$ ,  $\varphi_1 \rightarrow \varphi_2 \equiv \neg\varphi_1 \vee \varphi_2$ ,  $\varphi_1 \leftrightarrow \varphi_2 \equiv (\varphi_1 \rightarrow \varphi_2) \wedge (\varphi_2 \rightarrow \varphi_1)$ ,  $\varphi_1 \otimes \varphi_2 \equiv (\varphi_1 \wedge \neg\varphi_2) \vee (\neg\varphi_1 \wedge \varphi_2)$ ,  $Bif_a\varphi \equiv B_a\varphi \vee B_a\neg\varphi$ .

A formula of our language without any modality is said to be *objective*.

We now give an axiom system for belief and intention. This axiom system consists of following reasoning rules. Let  $a$  be an agent of  $\mathcal{A}$ .

- Propositional tautologies
- KD45 pour  $B$ ,

$$(K) B_a(\varphi \rightarrow \psi) \wedge B_a\varphi \rightarrow B_a\psi$$

$$(D) B_a\varphi \rightarrow \neg B_a\neg\varphi$$

$$(4) B_a\varphi \rightarrow B_aB_a\varphi$$

$$(5) \neg B_a\varphi \rightarrow B_a\neg B_a\varphi$$

- (UE) Unit exclusion for  $I_a, \neg I_a(\top)$
- *BI* Introspection as follows,

$$(BI1) I_a\varphi \rightarrow B_aI_a\varphi$$

$$(BI2) \neg I_a\varphi \rightarrow B_a\neg I_a\varphi$$

$$(BI3) B_a(\varphi \leftrightarrow \psi) \rightarrow (I_a\varphi \leftrightarrow I_a\psi)$$

Inference rules are Modus Ponens (MP) and Necessitation for  $B_a$  (Nec), i.e.  $\frac{\varphi}{B_a\varphi}$ .

For belief modality, we suppose that agent do not have inconsistent beliefs (D) and that are conscious of what they believe (4) and what they do not believe (5).

For intention modality, we suppose that agents cannot intend a tautology to be true (UE).

Finally, we suppose some relation between belief and intention that we call belief intention introspection. First, we suppose that agents are conscious of what they intend (BI1) and what they do not intend (BI2). Then, we suppose that if an agent believes that two propositions are equivalent, then intending the first one to be true is equivalent to intending the second one to be true (BI3).

From this axiom system, we can derive the following theorem:  $I_a\varphi \rightarrow \neg B_a\varphi$ . It means that if an agent intends a proposition to be true, then it does not believe that this proposition is true. In other words, agents cannot intend what they already believe to be true. Thus, the framework suggested here is very close to [14] where intention and belief are defined in order to study notions of cooperation and speech acts.

The semantics for intention is inspired by [13]. We consider frames that are the hybrid of neighborhood frames (for intention modality) and Kripke structure (for belief modality). We have shown that the axiom system defined previously is sound and complete in respect to serial, transitive, euclidean, introspective and unit-exclusive models.

### 3 Relevance

In this section, we first introduce a formal definition for agent-oriented relevance. Then, we study its properties.

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### 3.1 Definition

We define relevance the following way:

**Definition 2** Let  $a$  be some agent of  $\mathcal{A}$ ,  $\varphi$  a formula and  $Q$  a request.  $\varphi$  is said to be relevant for agent  $a$  concerning request  $Q$  iff the following formula is true

$$I_a Bif_a Q \wedge (B_a(\varphi \rightarrow Q) \otimes B_a(\varphi \rightarrow \neg Q)) \wedge \varphi$$

This formula is denoted  $R_a^Q \varphi$ .

This definition comprises three elements:

- **Agent's information need**  $I_a Bif_a Q$ : We suppose that the agents that exchange pieces of information have some information needs. Moreover, we suppose that an information need is quite simple and can be modelled the following way: "agent  $a$  wants to know if  $Q$  or if  $\neg Q$ ,  $Q$  being a request".<sup>1</sup> Formally, information need is written  $I_a Bif_a Q$ , that means agent  $a$  wants to know if  $Q$ .
- **Agent's beliefs**  $B_a(\varphi \rightarrow Q) \otimes B_a(\varphi \rightarrow \neg Q)$ : Using its beliefs and the piece of information  $\varphi$ , the agent must be able to answer her request  $Q$ , that means she can deduce either  $Q$  or  $\neg Q$ . In order to represent this deduction, we choose logical implication. If some agent, from a piece of information  $\varphi$  can deduce both  $Q$  and  $\neg Q$ , then  $\varphi$  does not really answer the information need. Using  $\otimes$  prevents this case to happen<sup>2</sup>.
- **The piece of information truth value**  $\varphi$ : We consider that a false piece of information cannot be relevant. A false piece of information, even it has a meaning, is false. Misinformation is deleterious in a cooperative context. For example, let us consider an agent, that wants to take a train to Paris. This train leaves at 1.05 pm. In this context, telling the agent that its train leaves at 1.15 pm is damaging (as it can miss its train). Then, we cannot consider that the piece of information "The train leaves at 1.15 pm" is relevant to the agent.<sup>3</sup>

The following example illustrates the definition of relevance.

**Example 1** Let us consider two robots  $a$  and  $b$  that have to go into some direction. Unfortunately, some obstacle can block their ways (modelled by obstacle).

Robot  $a$  needs to know whether or not there is an obstacle. Its information need is modelled by  $I_a(B_a \text{obstacle} \otimes B_a \neg \text{obstacle})$ .

Let us consider the piece of information "there is an explosive" modelled by  $exp$ . Let us suppose that this is true.

Robot  $a$  believes that if there is an explosive, then there is an obstacle. This is modelled by  $B_a(exp \rightarrow \text{obstacle})$ .

Thus, in this case, we have:

- $I_a Bif_a(\text{obstacle})$
- $B_a(exp \rightarrow \text{obstacle})$
- $exp$

<sup>1</sup>In this paper, we do not pay attention to transitions from individual goals to information needs and from information need ( as it is perceived by the agent ) to a formalized request.

<sup>2</sup>Using  $\otimes$  prevents the case where the agent already believes  $\neg\varphi$  to happen. Indeed, in this particular case, from  $\neg\varphi$ , the agent would be able to deduce anything.

<sup>3</sup>In some particular cases, misinformation can be relevant. For example, it is relevant for a teacher to learn that one of his pupils is wrong about some lessons. However, in this case, this is not the wrong lesson itself that is relevant to the teacher but the fact that the pupil is wrong. And this fact is true.

- Then, we can deduce  $R_a^{obstacle}(exp)$

That means that information  $exp$  is relevant to robot  $a$  concerning its request  $obstacle$ . Indeed, from  $exp$  which is a true piece of information, robot  $a$  can deduce that there is an obstacle in its direction.

Robot  $b$  also needs to know whether or not there is an obstacle. Its beliefs are different from  $a$ 's ones. Indeed, robot  $b$  believes that if there is no explosive, then there is no obstacle (meaning that explosive are the only objects considered as obstacle). This can be modelled by  $B_b(\neg exp \rightarrow \neg obstacle)$ <sup>4</sup>. Thus, we have :

- $I_b B_i f_b obstacle$
- $B_b(\neg exp \rightarrow \neg obstacle)$
- $exp$

Then, the piece of information  $exp$  is not relevant for robot  $b$  as it cannot deduce neither  $obstacle$  nor  $\neg obstacle$ .

The information  $\neg exp$ , which is false, cannot be relevant for robot  $b$  as it would allow  $b$  to make wrong conclusions.

### 3.2 Properties

In this part, we study some properties of the relevance operator. For that, let us take an agent  $a$  of  $\mathcal{A}$ ,  $Q$ ,  $Q_1$  and  $Q_2$  some requests,  $\varphi$ ,  $\varphi_1$ ,  $\varphi_2$  some formulas. The following propositions are theorems of our logic<sup>5</sup>.

**Proposition 1**  $R_a^Q \varphi \rightarrow \neg B_a \varphi \wedge \neg B_a \neg \varphi$

If some piece of information  $\varphi$  is relevant for some agent  $a$ , then agent  $a$  does not believe neither  $\varphi$  (otherwise she would be already able to answer its information need), or  $\neg \varphi$  (in contradiction with the use of  $\otimes$  in definition of relevance).

**Proposition 2** •  $I_a B_i f_a Q \rightarrow R_a^Q Q \otimes R_a^Q \neg Q$  : one of the pieces of information  $Q$  or  $\neg Q$  is relevant to agent  $a$  concerning its request  $Q$ .

- $R_a^Q \varphi \leftrightarrow R_a^{\neg Q} \varphi$  : some piece of information that is relevant concerning a request  $Q$  is relevant too concerning the request  $\neg Q$ .
- $\neg(\varphi_1 \wedge \varphi_2) \rightarrow \neg(R_a^{Q_1} \varphi_1 \wedge R_a^{Q_2} \varphi_2)$  : two conflicting pieces of information cannot both be relevant.

**Proposition 3**  $R_a^Q \varphi \rightarrow \neg B_a R_a^Q \varphi$

If some information  $\varphi$  is relevant to some agent  $a$ , then  $a$  does know it. This is due to the truth value of the piece of information contained in the relevance definition. If the agent believes that the piece of information is relevant to it, then it believes this piece of information. If it believes this piece of information, then it can deduce from its set of beliefs the answer to its information need. This is in contradiction with the fact the agent has the information need (relation of strong realism between belief and intention).

**Proposition 4** Let  $*$  be some belief revision operator satisfying AGM postulates [15].  $Bel_a$  represents the set of beliefs of agent  $a$  and  $Bel_a * \varphi$  the set of beliefs of agent  $a$  after being revised by  $\varphi$  using revision operator  $*$ . Then, we have  $(R_a^Q \varphi \rightarrow (Bel_a * \varphi \rightarrow Q)) \otimes (R_a^Q \varphi \rightarrow (Bel_a * \varphi \rightarrow \neg Q))$

This proposition shows that the deduction operator that we have chosen, logical implication, corresponds to some “basic” belief revision operator. Indeed, if it revises its beliefs with the relevant piece of information, the agent has in its new beliefs set the answer to its information need.

**Notation.** In what follows, we will write  $B_a(\varphi_1, \varphi_2/Q)$  instead of  $\neg(B_a(\varphi_1 \rightarrow Q) \wedge B_a(\varphi_2 \rightarrow \neg Q)) \wedge \neg(B_a(\varphi_1 \rightarrow \neg Q) \wedge B_a(\varphi_2 \rightarrow Q))$ . This formula means that agent  $a$  believes that  $\varphi_1$  and  $\varphi_2$  do not allow to deduce a contradiction concerning  $Q$ .

<sup>4</sup>We also suppose that robot  $b$  does not have any other belief about  $exp$  or  $obstacle$

<sup>5</sup>To lighten, we will not write the symbol  $\vdash$  in front of theorems.

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**Proposition 5**  $B_a(\varphi_1, \varphi_2/Q) \rightarrow (\varphi_2 \wedge R_a^Q \varphi_1 \rightarrow R_a^Q(\varphi_1 \wedge \varphi_2))$

**Proposition 6**  $B_a(\varphi_1, \varphi_2/Q) \rightarrow (R_a^Q \varphi_1 \wedge R_a^Q \varphi_2 \rightarrow R_a^Q(\varphi_1 \vee \varphi_2))$

Those two propositions show that the relevance operator characterizes too many relevant pieces of information. this is illustrated in the following example.

**Example 2** *Let us continue example 1. Robot a needs to know whether or not there is an obstacle and we suppose that  $exp$  is relevant to it.*

*Let us suppose that the piece of information “it rains”, modelled by  $rain$  is true in this context. Then, the piece of information  $exp \wedge rain$  is relevant to a. Indeed, it contains all necessary elements so that robot a is able to answer its information need. Nevertheless, intuitively, the piece of information  $exp$  is more relevant to a than  $exp \wedge rain$  because this last one contains the element  $rain$  that is not necessary to answer a’s information need.*

All the pieces of information characterized relevant are “sufficiently” relevant. Indeed, each of them gives an answer to the information need. On the other side, one could consider pieces of information that are “necessarily” relevant, that means the ones without which the agent cannot answer its information need. If we combine the two concepts, we can find, among the “sufficiently relevant” pieces of information, the ones that are the most “necessary”. Thus, those most necessary pieces of information are the very ones that are the most relevant.

## 4 A hierarchy for relevance

### 4.1 Minimal explanation

In this section, we characterize the “necessary relevance” notion described below. For that, we first introduce the notion of minimal explanation. This notion has been used in Lakemeyer [5] to define relevance. However, the definition of minimal explanation he uses is quite syntactical<sup>6</sup>. In order to have a more semantic definition, we update the definition by using notions of semantic independence defined in [6].

**Definition 3** *Let  $\varphi$  be an objective formula.  $\varphi$  is said to be in Negation Normal Form (NNF) if and only if only propositional symbols are in the scope of an occurrence of  $\neg$  in  $\varphi$ .*

*$Lit(\varphi)$  denotes the the set of literals occurring in the NNF of  $\varphi$ .*

For example, the NNF form of  $\varphi = \neg((\neg p \wedge q) \vee r)$  is  $(p \vee \neg q) \wedge \neg r$ . Then,  $Lit(\varphi) = \{p, \neg q, \neg r\}$ .

**Definition 4** *Let  $\varphi$  be an objective formula,  $l$  a literal.*

*$\varphi$  is said to be syntactically Lit-dependent on  $l$  (resp. syntactically Lit-independent from  $l$ ) if and only if  $l \in Lit(\varphi)$  (resp.  $l \notin Lit(\varphi)$ ).*

**Definition 5** *Let  $\varphi$  be an objective formula,  $l$  a literal.*

*$\varphi$  is said to be Lit-independent from  $l$ , denoted  $l \not\mapsto \varphi$ , if and only if there exists a formula  $\Sigma$  such that  $\Sigma \equiv \varphi$  and  $\Sigma$  is syntactically Lit-independent from  $l$ . Otherwise,  $\varphi$  is said to be Lit-dependent on  $l$ , denoted  $l \mapsto \varphi$ . Given a language, the set of all literals of this language such that  $l \mapsto \varphi$  is denoted by  $DepLit(\varphi)$ .*

**Example 3** *Let  $\varphi = (p \wedge \neg q \wedge (p \vee q))$ . We have  $DepLit(\varphi) = \{p, \neg q\}$ . Note that  $\varphi$  is Lit-independent from  $q$  because it is equivalent to  $\Sigma = (p \wedge \neg q)$ , in which  $q$  does not appear positively.*

Now, let us give the definition of minimal explanation.

<sup>6</sup>Indeed, he uses CNF form of a formula. But for a given formula, the CNF form is not unique

**Definition 6** Let  $\Delta$  be a finite set of objective formulae, and  $\alpha$  and  $\beta$  be two objective formulae.  $\beta$  is an explanation of  $\alpha$  if and only if  $\vdash B\Delta \rightarrow B(\beta \rightarrow \alpha)$  and  $\nexists B\Delta \rightarrow B(\neg\beta)$ .  $\beta$  is a minimal explanation of  $\alpha$  if and only if  $\beta$  is an explanation of  $\alpha$  and there is no explanation  $\beta'$  of  $\alpha$  such that  $DepLit(\beta') \subseteq DepLit(\beta)$ .

## 4.2 Most relevant information

From this minimal explanation, we can define what are the most relevant formulae.

Let  $\mathcal{R}_a^Q$  be the set of relevant formulae. For all  $\varphi$  in  $\mathcal{R}_a^Q$ , we have  $B_a(\varphi \rightarrow Q)$  or  $B_a(\varphi \rightarrow \neg Q)$  and  $\neg B_a(\neg\varphi)$ , that means that for all  $\varphi$  in  $\mathcal{R}_a^Q$ ,  $\varphi$  is an explanation of  $Q$  or  $\neg Q$ .

**Definition 7** Let  $\mathcal{Rm}_a^Q$  be the subset of  $\mathcal{R}_a^Q$  that contains the minimal explanations of  $Q$  and  $\neg Q$ . We will write  $Rm_a^Q\varphi$  to express that the formula  $\varphi$  belongs to  $\mathcal{Rm}_a^Q$ .

**Example 4** Let us consider the following set of relevant pieces of information to agent  $a$  concerning its request  $Q$ :  $\mathcal{R}_a^Q = \{exp \wedge rain, exp \vee fire, fire\}$ . Then  $\mathcal{Rm}_a^Q = \{fire, exp \wedge rain\}$ .

Thus, necessary (in respect to minimal explanation) and sufficient relevant pieces of information can be characterized. Of course, according to a different definition of “necessary” for a piece of information, we could have a different set of most relevant pieces of information.

## 5 Cooperation

Let us come back to the notion of cooperation in communication. Now that we have given a formal definition for relevance, we can formally define the notion of cooperation. For that, we extend our logical framework and consider for every couple  $(a, b)$  of agents in  $\mathcal{A}$  the operator  $Inf_{a,b}$  defined in [16]<sup>7</sup>.  $Inf_{a,b}\varphi$  is read “agent  $a$  informs  $b$  about  $\varphi$ ”. This operator is a non-normal operator for which we only have the substitutivity of equivalent formulae.

$$\frac{\varphi \leftrightarrow \psi}{Inf_{a,b}\varphi \leftrightarrow Inf_{a,b}\psi}$$

Intuitively, cooperation expresses the fact that only relevant information are exchanged. This is formalized the following way :

**Definition 8** Let  $a$  and  $b$  be two agents of  $\mathcal{A}$ . The agent  $b$  is cooperative in regard to  $a$  iff for any formula  $\varphi$ ,  $b$  informs  $a$  about  $\varphi$  if and only if there is a request  $Q$  such that  $b$  believes that  $\varphi$  is maximal relevant for  $a$  concerning  $Q$ . This is represented<sup>8</sup> by :

$$Coop(b, a) \equiv \forall\varphi, Inf_{b,a}\varphi \leftrightarrow \exists Q, B_b(Rm_a^Q\varphi)$$

Thus, an agent is cooperative in regard to another if it informs the other agent about pieces of information that it thinks maximal relevant for it and only those pieces of information. In other words, the set of exchanged pieces of information from  $b$  to  $a$  is exactly the set of pieces of information that  $b$  believes to be maximal relevant for  $a$  concerning any of its needs.

Thus, with this definition for cooperation, an agent  $b$  is *non-cooperative* in regard to another agent  $a$

1. if  $b$  informs  $a$  about something for which  $b$  believes  $a$  has no need for
- or 2. if  $b$  believes that a piece of information is maximal relevant for  $a$  and does not inform  $a$  about it.

<sup>7</sup>This operator was denoted  $I_{a,b}$  in [16]

<sup>8</sup>The following formula cannot be represented in our framework. It is just a notation

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As we have seen, many pieces of information are relevant ( $exp, rain \wedge exp$  for example). This is why  $b$  should only inform  $a$  about what it thinks to be maximal relevant for  $a$ .

Many definitions of cooperation do exist in the literature [9, 17, 16]. In what follows, we compare the definition proposed here with two notions of cooperation: Sadek's [17] and Demolombe's [16].

Sadek [17] took a particular interest in studying the human-machine interaction. For him, being cooperative means giving back cooperative answers, i.e. answers that relevantly extend the question that was explicitly asked. Thus, a system can give back different types of cooperative answers to some user:

- completing answers: additional pieces of information that the user did not explicitly ask.
- correcting answers: pieces of information that invalidate some user's presuppositions.
- suggestive, conditional answers, ...

Sadek's approach contains some notions that are central in the characterisation of relevance and of cooperation put forward in this paper. Indeed, even if Sadek does not insist on this point, he considers that there is an implicit need of the user underlying his cooperation. This can be a need for additional pieces of information, or a need for correcting pieces of information and the type of the need will induce the type of the answer. Moreover, the fact that the system gives back a piece of information he possesses in its database is in accordance with our hypothesis. To conclude, it seems that our modelling framework is compatible with Sadek's approach about completing answers.

The cooperation defined in [16] is the closest one to the cooperation we define. This is why we compare the two notions. In [16], Demolombe defines the notion of cooperation from one agent in regard to another about a piece of information. We will represent agent  $b$  is cooperative in regard to agent  $a$  for piece of information  $p$  by  $C_p(b, a)$ . Following [16],

$$C_p(b, a) \equiv B_b p \leftrightarrow Inf_{b,a} p$$

That means that agent  $b$  is cooperative in regard to agent  $a$  for  $p$  iff if  $b$  believes  $p$  then  $b$  informs  $a$  about it.

The biggest difference between the Demolombe's definition and the one we propose is the presence of the information need. In [16], Demolombe does not take into account the information need of the receiver. Thus, this last one can receive pieces of information for which he has no interest or no need but that are true in some others agent belief base and for which this agent will be cooperative for.

In the definition we propose for cooperation, information exchanged from  $b$  to  $a$  should not only be believed by  $b$  but must also be believed by  $b$  as most relevant for  $a$ .

Let us illustrate the two cooperation definitions on an example.

**Example 5** *Let's consider again two robots  $a$  and  $b$ . Robot  $a$  needs to know whether or not there is an obstacle in its area and robot  $b$  knows that. Robot  $b$ , which has access to a map on which explosives are indicated, believes that there is an explosive on  $a$ 's area and that this explosive is of type  $T$ . Then we have:*

- $B_b(I_a B_i f_a obstacle)$
- $B_b(explosive) \wedge B_b(T)$
- $B_b(B_a(explosive \rightarrow obstacle))$

*We can conclude:*

- $B_b Rm_a^{obstacle} explosive$

*I.e,  $b$  thinks that the most relevant piece information for  $a$  is "there is an explosive".*

1. *Let us assume that  $b$  tells  $a$  that there is an explosive, i.e consider  $Inf_{b,a} explosive$ . Then:*

- *$b$  is cooperative in regard to  $a$ , i.e we have  $Coop(b, a)$  because  $b$  believes that explosive is relevant for agent  $a$  and this is the only information exchanged.*
- *$b$  is also cooperative in respect to agent  $a$  for explosive i.e  $C_{explosive}(b, a)$ .*



- However,  $b$  is not cooperative in respect to agent  $a$  for  $T$  i.e.  $\neg C_T(b, a)$  because  $b$  believes it but does not tell  $a$ .
2. Now, let us suppose that  $b$  tells  $a$  that there is an explosive, i.e.  $\text{Inf}_{b,a} \text{explosive}$  and also tells it that this explosive is of type  $T$ , i.e.,  $\text{Inf}_{b,a} T$ . In this case,
- $b$  is cooperative in respect to agent  $a$  for explosive and for  $T$  i.e.  $C_{\text{explosive}}(b, a)$  and  $C_T(b, a)$ .
  - However,  $b$  is not cooperative in regard to  $a$ , i.e.  $\neg \text{Coop}(b, a)$  because there is an information that is exchanged and that is not believed by  $b$  to be relevant for  $a$ .  $a$  has no need about  $T$  so it should not be informed about it.

This example shows that our definition of cooperation defines what information should be exchanged whereas Demolombe's one does not consider this point.

## 6 Conclusion

In multi-agent systems in which agents are grouped in order to achieve a global goal none of them would be able to achieve alone, like systems of systems or coalitions, agents have to cooperatively act so that the achievement of their individual goals ensures the achievement of the global goal. In particular, they have to cooperatively communicate and thus exchange information which is the very one useful. We call them relevant.

In this paper, we provided a formal definition of the notion of relevance. Given an agent that has some information need, we expressed in a multi-modal framework, what are relevant pieces of information for it concerning its information need. As too many pieces of information are relevant, we proposed a hierarchy for relevant pieces of information. This hierarchy can be seen as a compromise between "being precise" and "being concise".

From this characterization of relevance, we defined the notion of cooperation between agents that communicate. Thus, an agent is cooperative to another one if and only if it informs the other about and only about what it thinks maximal relevant for the other.

This work can be extended in many ways.

First, in the same way that we have defined a relevance concerning an information need, we could define a relevance concerning a verification need. In that case, any piece of information in accordance or in contradiction with the agent's beliefs (in a given domain) would be relevant. For example, a robot can believe that there is no explosive in his direction whereas there is an explosive. In that case, the information that there is an explosive is relevant to this robot. Thus, it would be possible to define a new cooperation according to this relevance. This cooperation would correspond to Sadek's notion of correcting answers.

In the same idea, it would be possible to define a relevance concerning a completeness need. In that case, any piece of information that matches some characteristic defined by the agent is relevant for it. For example, an agent could want to be informed of any information (that it doesn't know yet) about some area he is responsible of.

Finally, we could extend this present work by considering the notion of time. Indeed, information need, truth value of a piece of information or beliefs are concepts that change with time. For example, a robot  $a$  needs to know before date  $D_0$  whether or not there is an obstacle. Another robot  $b$  believes that there is an obstacle from date  $D_1$  to date  $D_2$ . And the piece of information "there is an obstacle" is true from date  $D_3$  to date  $D_4$ . In that case, when is the information believed by  $b$  to be relevant for  $a$ ? In the same idea, the issue of time should be considered in cooperation definition.

## Acknowledgments

Stéphanie Roussel's PhD is granted by DGA (Direction Générale de l'Armement) and Laurence Cholvy is granted by ANR-CSOSG (project CAHORS).

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